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FINAL SCIENTIFIC REPORT ON AFOSR-77-3290.(U)
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FINAL SCIENTIFIC REPORT
of work completed during the grant years
4/1/77 to 3/31/82
under AFOSR Grant No. 77-3290

Prepared by

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SECTION 1. Introduction.

This report is a summary of the research conducted by the principal investigator, Jay R. Walton, under AFOSR Grant 77-3290 for the years 4/1/77 to 3/31/82. Much of the work described in this report was in collaboration with Dr. Arje Nachman who was a co-principal investigator on the grant from 4/1/77 until 3/31/79. A variety of theoretical questions concerning contact and fracture phenomena for elastic and viscoelastic material and associated mathematical techniques were addressed under the grant. The particular topics studied were:

1. quasi-static contact problems for power-law linearly viscoelastic material;
2. the quasi-static propagation of a crack in a power-law linearly viscoelastic material;
3. systems of generalized Abel integral equations, and simultaneous dual integral equations;
4. the application of conservation integral techniques to static elastic interfacial edge cracks;
5. thermo-elastic interfacial cracks;
6. the dynamic, steady-state fracture of general linearly viscoelastic material.

A brief description of these projects and a summary of the principal research accomplishments achieved on each follows in the next section. More detailed discussions of the above problem areas appear in the various annual and technical reports that have been prepared under this grant. The final section of this report is a compilation of abstracts from the several journal publications that arose from the research supported by this grant.

SECTION 2. Summary of Research Activity.

1. The initial research effort under this AFOSR grant involved a study of the quasi-static sliding contact friction between two solid bodies, one of which is assumed to be rigid and the other of which is assumed to be modelled as a linearly viscoelastic solid. This class of problems has significant engineering application to the prediction of the frictional force between two bodies in sliding contact where one exhibits mechanical hysteresis (viscoelasticity) and the other is very much harder and more rigid. Two examples of such interactions are tires (rubber) against pavement (rock) and polymeric materials in contact with rigid objects (as occurs in many structural applications of, say, graphite-epoxy composite materials).

Sliding contact problems in linear viscoelasticity theory have received considerable attention by analysts during the last two decades. The primary objective of these studies has been to predict the dependence of the contact pressure distribution, friction coefficient (or mechanical dissipation) and contact area on the applied force, the speed of the slider and the viscoelastic properties of the other contacting body. In none of these studies was an exact analytical solution found, except for the case in which the material is characterized as a standard linear solid, i.e., the viscoelastic shear modulus is assumed to be a simple exponential function of time and Poisson's ratio is constant. This type of model is far too simple to represent actual viscoelastic data.

It has been observed experimentally that many viscoelastic materials can be characterized rather well with a simple power-law model, i.e., by assuming a constant Poisson's ratio and a shear modulus that is a power function of time. Consequently, it was deemed important to attempt an analysis of the problem of a rigid indentor sliding over a power-law linearly viscoelastic half-plane. It was also considered to be of both mathematical and physical interest to incorporate into the model a Coulomb frictional force between the indentor and the half-plane, i.e., under the slider the shearing tractions are proportional to the normal tractions.

A closed form solution was obtained to this problem by reduction to a generalized Abel type integral equation for the unknown normal traction under the slider. This equation was then solved by a transformation to a Riemann-Hilbert boundary value problem. All important physical parameters were calculated for three canonical indentor shapes: a parabolic punch (as an approximation to a cylinder), a wedge punch and a flat punch. This work appears in the paper: "The sliding of a rigid indentor over a power-law viscoelastic half-space," J. R. Walton, A. Nachman and R. A. Schapery, Quart. J. Mech. Appl. Math., (31) #3, 1978.

Subsequently, the problem of a rigid indentor sliding over a power-law viscoelastic layer of finite thickness was considered. Mathematically, the layer problem is much more difficult. Consequently, a thick layer asymptotic analysis approach was adopted. This work appears in the paper: "The sliding of a rigid indentor over a power-law viscoelastic layer," A. Nachman and J. R. Walton, J. Appl. Mech. (100) #1, 1978.

2. Subsequent to the work on friction, attention was focused on a study of the fracture of viscoelastic material. Techniques similar to those employed in the quasi-static contact analysis were used to obtain an analytical solution to the problem of the propagation by a rigid wedge of a 2-dimensional crack in an infinite power-law linearly viscoelastic material. This work appears in the paper: "The propagation of a crack by a rigid wedge in an infinite power-law viscoelastic body," J. R. Walton and A. Nachman, J. Appl. Mech., (46) #3, 1979.

The content and significance of this study is perhaps most succinctly summarized in the following excerpt from the cited paper.

> This particular problem is an example of a physically realizable mode of fracture to which the correspondence principle does not apply. Moreover, the close-dorm solutions obtained here provide a method for experimentally determining physical parameters, such as the stress-intensity factor, important for an understanding of fracture phenomena.
>
> This problem has been studied extensively for elastic material, a detailed description of which appears in the article by Barenblatt [2]. The wedging problem was considered by Barenblatt for two reasons. In general, Barenblatt addressed himself to the issues of unrealistic singular crack tip stresses and indeterminant crack lengths encountered in fracture in classical linear elasticity. The wedge problem exhibits both features even though the crack is "semi-infinite" since the free crack surface ahead of the wedge is finite. In particular, the thrust of Barenblatt's article is that in a neighborhood of the crack tip, the separated surfaces experience cohesive molecular forces which produce a cusp profile and a finite stress field. Barenblatt accounts for this by the phenomenological expedient of assuming that for equilibrium cracks a cohesive normal traction acts over a small interval behind the crack tip with a constant stress-intensity factor characteristic of the material. As noted by Barenblatt, the finite crack length is determined by merely

knowing this stress-intensity factor and is independent of the actual distribution of cohesive forces. The utility of this observation lies in reversing the implication. Namely, if experimentally a wedge is driven into an elastic material and the length of the free surface of the resultant equilibrium crack is measured, then the stress-intensity factor may be easily calculated.

In this paper, we provided simple closed-form expressions for the stress and displacement fields for the quasi-static wedging of a power law viscoelastic material under the Barenblatt hypothesis. This analysis culminates in a relation among the material parameters, the material stress-intensity factor and the length of the free crack surface.

3. Concurrent with the viscoelastic fracture effort, a study of a broad class of simultaneous systems of generalized Abel integral equations was initiated and completed. This study was motivated in part by the appearance of generalized Abel equations in the power-law viscoelastic contact and fracture work and in part by the fact that such systems were known to arise in the analysis of certain simultaneous systems of dual integral equations. These systems of dual integral equations occupy a central role in the analysis of elastic bi-media (interfacial) cracks. This work is contained in a series of two papers: "Systems of generalized Abel equations," M. Lowengrub and J. R. Walton, SIAM J. Math. Anal. (10)#4, 1979; "Systems of generalized Abel integral equations with applications to simultaneous dual relations," J. R. Walton, ibid.

4. Further questions on elastic interfacial cracks were addressed. Specifically, the problem of determining the energy release rate (a quantity necessary for a prediction of the ultimate strength of a cracked solid) for elastic interfacial edge cracks was considered. The associated boundary value problem is very difficult to solve explicitly. However, by use of the M-conservation integral [illegible]

by L. B. freund in the calculation of stress-intensity factors for cracks in homogeneous elastic media), closed form expressions for the desired energy release rate were obtained. The technique was adapted to both a mode III and a mixed modes I and II analysis. Part of this work was completed while the principal investigator, J. R. Walton, was a Visiting Associate Professor with the Mathematics Research Center at the University of Wisconsin, and appears in the MRC Technical Symmary Report #2017. Moreover, the project resulted in the two papers: "Energy release rate calculations for interface edge cracks based on a conservation integral, " A. Nachman and J. R. Walton, Int. J. Solids. Struct. (16), 1980; "Energy release rate calculations for an interface Mode III edge crack based on a conservation integral," A. Nachman, J. Tweed and J. R. Walton, Lett. Appl. Enging. Sci., (19), 1981.

5. In collaboration with Professor M. Lowengrub of the Department of Mathematics, Indiana University, a study of thermo-elastic interfacial cracks was initiated. Specifically, work is in progress and nearly completed on the determination of the combined thermo-mechanical stresses induced by a penny-shaped crack at the interface of two bonded dissimilar elastic half-spaces. The crack is forced open by a remotely imposed heat flux. Such a study has important engineering applications to an analysis of the strength of structures involving composite materials under non-isothermal conditions. This work will appear in a paper that is soon to be written and submitted for publications.

6. As part of a collaborative research effort with Professor R. A. Schapery of the Departments of Aerospace and Civil Engineering, Texas A&M University, a study of the dynamic fracture of general linearly viscoelastic material was initiated. Moreover, dynamic viscoelastic fracture is now the principal focus of research under the current AFOSR grant of the principal investigator, J. R. Walton. Three separate dynamic crack problems have been considered; each is described below.

A. An analysis of the anti-plane strain problem of a steadily propagating, semi-infinite crack in an infinite linearly visco-elastic body which is subjected to a distribution of shearing tractions moving with the crack was completed. This work is contained in the paper: "On the steady state propagation of an anti-plane shear crack in an infinite general linearly viscoelastic body," J. R. Walton, Quart. Appl. Math., April, 1982. The significance of these results is summarized in the 1980-81 Annual Scientific Reports.

B. An analysis was completed of the dynamic Mode III crack problem for a layer of finite thickness analogous to that described in paragraph A for an infinite body. A closed form expression for the stress intensity factor was obtained for very general visco-elastic shear modulus. The solution and analysis are much more complicated for a layer than for an infinite body, but the Riemann-Hilbert method that was employed yields a form for the stress intensity factor that illuminates the combined effects of material inertia and boundary interactions. A report on this work is currently being written for publication.

C. A study of the steady-state, dynamic Mode I crack problem fo an infinite general linearly viscoelastic body analogous to the Mode III problem discussed in paragraph A was initiated. The Mode I model is more important to engineering applications than the Mode III, but introduces significant additional mathematical complications. Only preliminary results have been obtained to date and these are described in the renewal proposal of the principal investigator that was submitted in December, 1981.

SECTION 3. List of Abstracts.

Walton, J. R., A. Nachman and R. A. Schapery, "The sliding of a rigid indentor over a power-law viscoelastic half-space," Quart. J. Mech. Appl. Math., (31) #3, 1978.

Summary--Closed form solutions are obtained for the problem of a rigid asperity sliding with Coulomb friction over a power-law viscoelastic half-space. The dual integral equations relating the unknown normal traction under the contact interval (also unknown) to the unknown normal displacement outside the contact interval are solved by first reducing the system to a generalized Abel integral equation and then appealing to the theory of Riemann-Hilbert boundary-value problems. The physical quantities of interest (eg. the coefficient of sliding friction) are determined for the three canonical indentors: a parabolic punch, a wedge punch, and a flat punch. The analysis predicts singularities in the normal stress field for certain power-law materials even for the smooth parabolic indentor.

Nachman, A. and J. R. Walton, "The sliding of a rigid indentor over power-law viscoelastic layer," J. Appl. Mech. (100) #1, 1978.

Summary--The problem of the sliding of a rigid asperity over a power law viscoelastic layer is examined in the realistic limit of infinite (dimensionless) layer thickness. For a contact interval of unit length, asymptotic expansions for the normal traction over the interval together with several other physically relevant quantities (e.g., the friction coefficient) are developed in terms of an appropriate asymptotic sequence of powers of the (dimensionless) layer thickness.

Walton, J. R. and A. Nachman, "The propagation of a crack by a rigid wedge in an infinite power law viscoelastic body," J. Appl. Mech., (46) #3, 1979.

Summary--Closed-form solutions are obtained for the problem of a crack propagated by a 2-dimensional rigid wedge of finite thickness in an infinite power law viscoelastic body. The distance from the crack tip to the point of contact with the wedge is determined from the assumption that in the vicinity of the crack tip, forces of cohesion act to produce a smoothly closing crack with continuous stresses. Finally, a simple formula expressing the stress-intensity factor as a function of the speed of penetration and the length of the free crack surface is exhibited.

Lowengrub, M. and J. R. Walton, "Systems of generalized Abel Equations," SIAM J. Math. Anal. (10) #4, 1979.

Abstract--Certain mixed boundary value problems arising in the classical theory of elasticity lead to the solution of certain systems of generalized Abel integral equations. A method is presented where these systems are reduced to uncoupled pairs of Riemann boundary value problems. Closed form solutions are obtained. We also demonstrate how general systems of dual relations (given in terms of Erdélyi-Sneddon operators of fractional integration) may be reduced to these systems of Abel equations.

Walton, J. R., "Systems of generalized Abel integral equations with applications to simultaneous dual relations," SIAM J. Math. Anal. (10) #4, 1979.

Abstract--A method is presented for solving certain systems of generalized Abel integral equations by constructing an equivalent system of singular integral equations. An application is then given to a class of simultaneous dual relations of a type arising in bimedia fracture problems in elasticity. The equations discussed in this paper generalize those considered in an earlier paper of Lowengrub and Walton [SIAM J. Math. Anal., this issue, pp. 794-807].

Nachman, A. and J. R. Walton, "Energy release rate calculations for interface edge cracks based on a conservation integral," Int. J. Solids. Struct. (16), 1980.

Abstract--A certain conservation integral, the so called M-integral, recently exploited by L. B. Freund in the calculation of stress intensity factors for cracks in homogeneous elastic media, is applied to the calculation of energy release rates for interface edge cracks. Specifically, for an edge crack along the interface between two elastic wedges of different opening angles and dissimilar elastic properties, and that is subjected to point loads at the apex, a relation is derived among the length of the crack, the energy release rate of the crack, the applied loads, the wedge angles and the material parameters.

Nachman, A., J. Tweed and J. Walton, "Energy release rate calculations for an interface Mode III edge crack based on a conservation integral," Lett. Appl. Engng. Sci., (19), 1981.

Abstract--The M-integral is applied to the calculation of energy release rates for interface edge cracks of the Mode III type. Specifically, for an edge crack along the interface between two elastic wedges of different opening angles and dissimilar elastic properties, and that is subjected to point loads at the apex, a relation is derived among the length of the crack, the energy release rate of the crack, the applied loads, the wedge angles and the material parameters.

Walton, J. R., "On the steady-state propagation of an anti-plane shear crack in an infinite general linearly viscoelastic body," Quart. Appl. Math., April, 1982.

Abstract--The steady-state propagation of a semi-infinite anti-plane shear crack is considered for a general infinite homogeneous and isotropic linearly viscoelastic body. Inertial terms are retained and the only restrictions placed on the shear modulus are that it be positive, continuous, decreasing and convex. For a given integrable distribution of shearing tractions travelling with the crack, a simple closed-form solution is obtained for the stress intensity factor and for the entire stress field ahead of and in the plane of the advancing crack. As was observed previously for the standard linear solid, the separate considerations of two distinct cases, defined by parameters c and c^*, arises naturally in the analysis. Specifically, c and c^* denote the elastic shear wave speeds corresponding to zero and infinite time, and the two cases are (1) $0 < v < c^*$ and (2) $c^* < v < c$, where v is the speed of propagation of the crack. For case (1) it is shown that the stress field is the same as in the corresponding elastic problem and is hence independent of v and all material properties, whereas, for case (2) the stress field depends on both v and material properties. This dependence is shown to be of a very elementary form even for a general viscoelastic shear modulus.

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This report summarizes progress on several theoretical questions concerning contact and fracture phenomena for elastic and viscoelastic material. Specifically, the following topics are discussed: (1) quasi-static sliding contact problems for power-law linearly viscoelastic material; (2) the quasi-static propagation of a crack by a rigid wedge through an infinite power-law linearly viscoelastic body; (3) solution techniques for systems of generalized Abel integral equations with application to simultaneous sets of dual (CONTINUED)

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ITEM #20, CONTINUED: integral equations and interfacial cracks in elastic material; (4) the application of conservation integral techniques to the calculation of energy release rates for elastic interfacial edge cracks; (5) static thermo-elastic interfacial edge cracks; and (6) the dynamic, steady-state fracture of general linearly viscoelastic material.